

A Numerical Investigation for Under Water Fluid- Netting Interaction Problem

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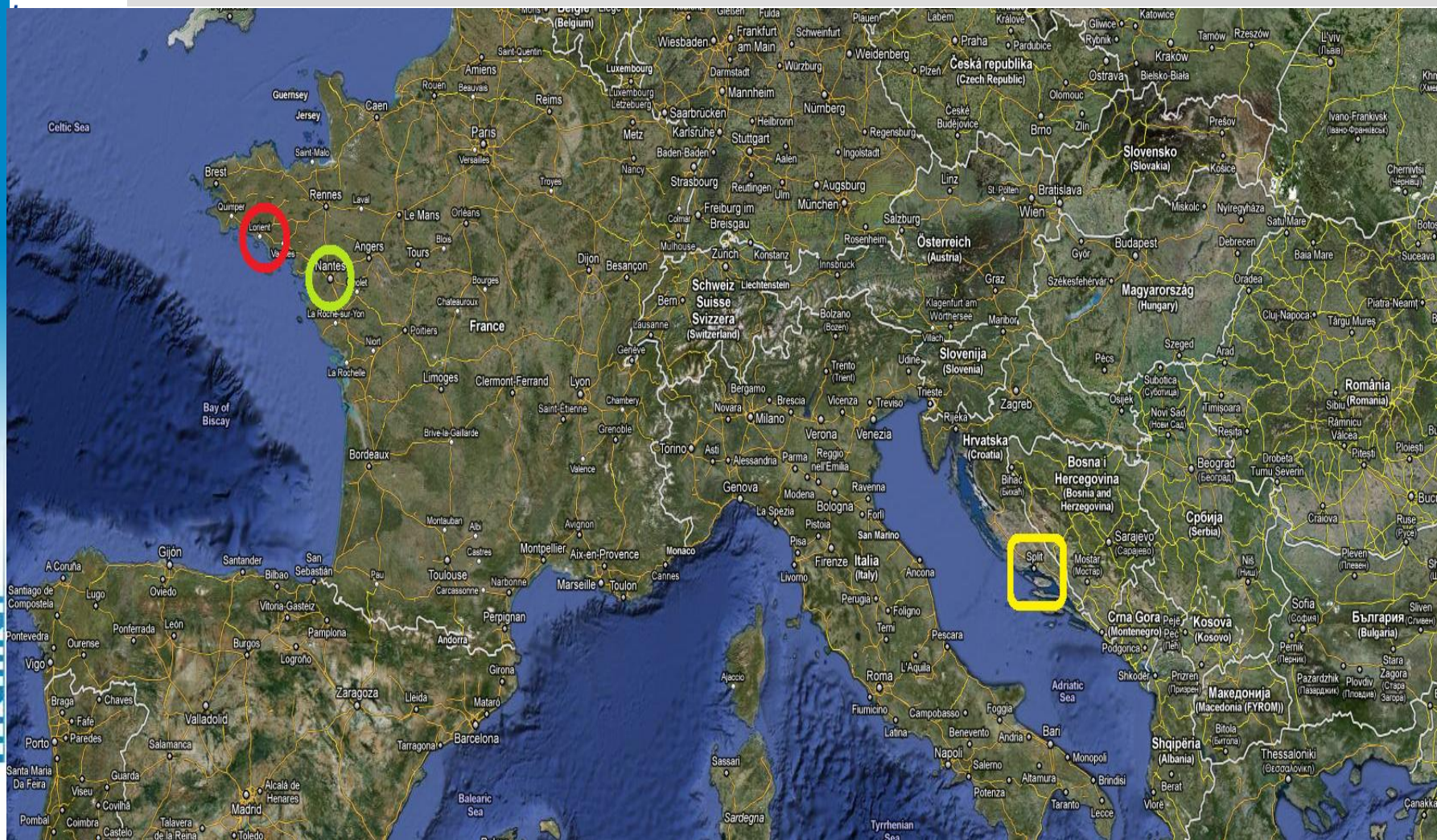
Outline

- **Motivation & Aims**
- **Proposed solutions**
- **Modeling and simulation**
- **Results**
- **Conclusions and outlook**

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Aims

Create a 3D CFD fluid dynamic code based on C++, to predict the water flow velocity field and couple it with the IFREMER structure code to ameliorate the trawl net deployment prediction.

Solutions

Direct solution : Represent the structure by connections of cylinders and spheres and introduce it into the fluid code by an obstacle, but due to the large number of net meshes it is currently not possible to make a direct discretization of the flow around every mesh structure and their boundary layers.

Approximate solution : Introduce the hydrodynamic forces calculated by the structure code on the assembly of the net, into the Navier Stokes code as source terms. This will reduce notably the computation time with preserving a good accuracy.

Modeling and Simulations

In order to work with an explicit time advancement, we choose the pseudo compressible method for the Navier Stokes equations.

$$\text{Mass: } \frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{u}) = 0$$

$$x\text{-momentum: } \frac{\partial(\rho u)}{\partial t} + \text{div}(\rho u \mathbf{u}) = -\frac{\partial p}{\partial x} + \text{div}(\mu \text{ grad } u) + S_{M_x}$$

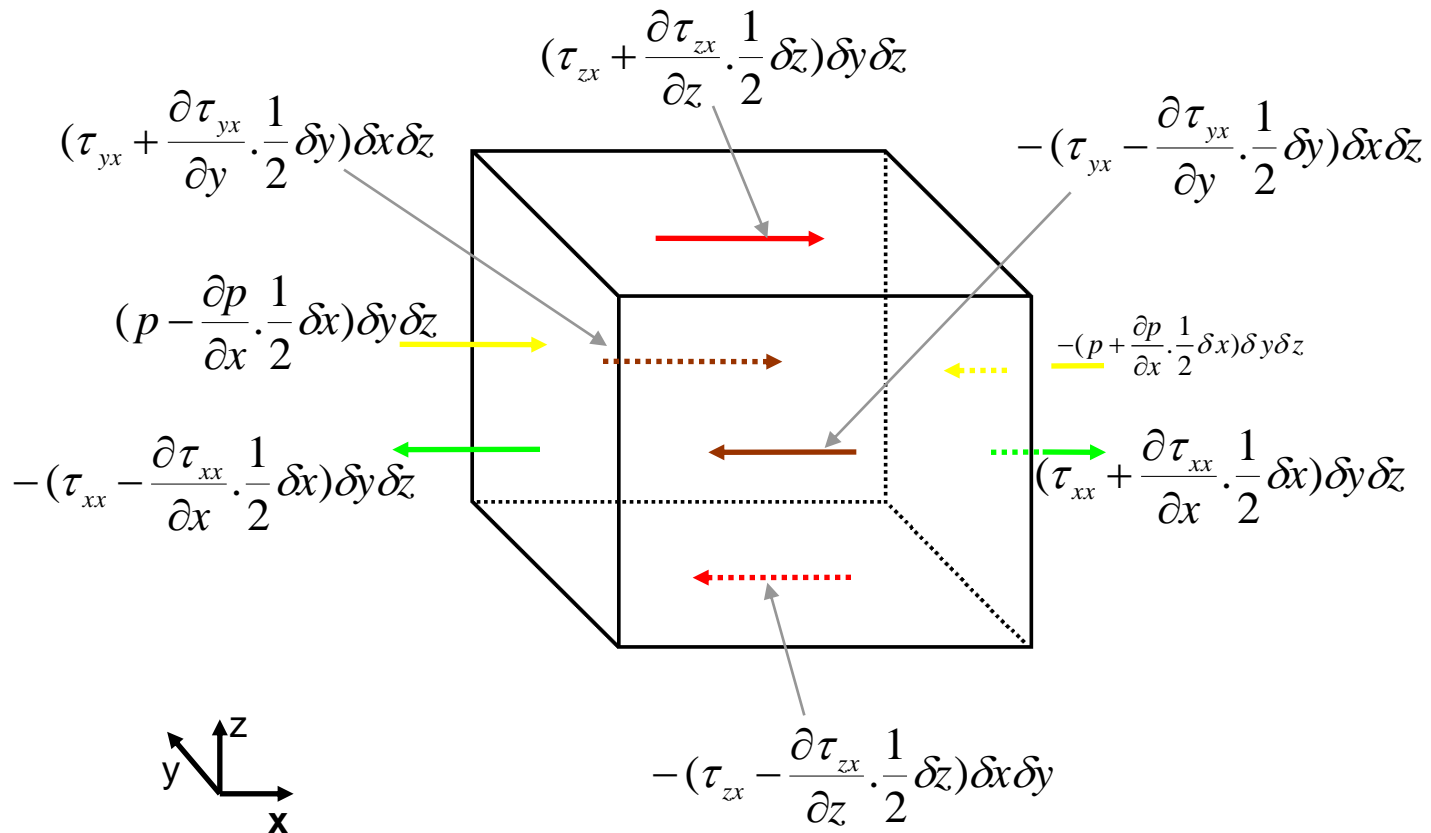
$$y\text{-momentum: } \frac{\partial(\rho v)}{\partial t} + \text{div}(\rho v \mathbf{u}) = -\frac{\partial p}{\partial y} + \text{div}(\mu \text{ grad } v) + S_{M_y}$$

$$z\text{-momentum: } \frac{\partial(\rho w)}{\partial t} + \text{div}(\rho w \mathbf{u}) = -\frac{\partial p}{\partial z} + \text{div}(\mu \text{ grad } w) + S_{M_z}$$

$$\text{Internal energy: } \frac{\partial(\rho i)}{\partial t} + \text{div}(\rho i \mathbf{u}) = -p \text{ div } \mathbf{u} + \text{div}(k \text{ grad } T) + \Phi + S_i$$

Modeling and Simulations

Forces acting on control volume of fluid. (Pressures and shear stress)



Modeling and Simulations

Conservative presentation of the inviscid Navier Stokes equation.

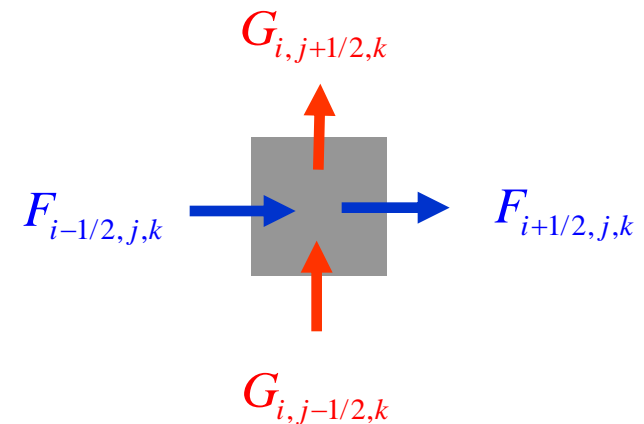
$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} + \frac{\partial G(U)}{\partial y} + \frac{\partial H(U)}{\partial z} = 0$$

$$U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E \end{pmatrix} \quad F = \begin{pmatrix} \rho u \\ \rho u^2 + P \\ \rho uv \\ \rho uw \\ u(E + P) \end{pmatrix} \quad G = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + P \\ \rho vw \\ \mu(E + P) \end{pmatrix} \quad H = \begin{pmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho w^2 + P \\ w(E + P) \end{pmatrix}$$

Modeling and Simulations

Finite volume method : We present the application of finite volume method to the compressible Navier-Stokes equations. The integration of the previous equation over a rectangular cell gives :

$$\iint_{\Omega_{ij}} \frac{\partial U}{\partial t} + \iint_{\Omega_{ij}} \left(\frac{\partial F(U)}{\partial x} + \frac{\partial G(U)}{\partial y} \right) = \iint_{\Omega_{ij}} J$$



$$U_{i,j,k}^{n+1} = U_{i,j,k}^n + \frac{\Delta t}{\Delta x} \left[F_{i+1/2,j,k}^{n+1/2} - F_{i-1/2,j,k}^{n+1/2} \right] + \frac{\Delta t}{\Delta y} \left[G_{i,j+1/2,k}^{n+1/2} - G_{i,j-1/2,k}^{n+1/2} \right] + \frac{\Delta t}{\Delta z} \left[H_{i,j,k+1/2}^{n+1/2} - H_{i,j,k-1/2}^{n+1/2} \right]$$

Modeling and Simulations

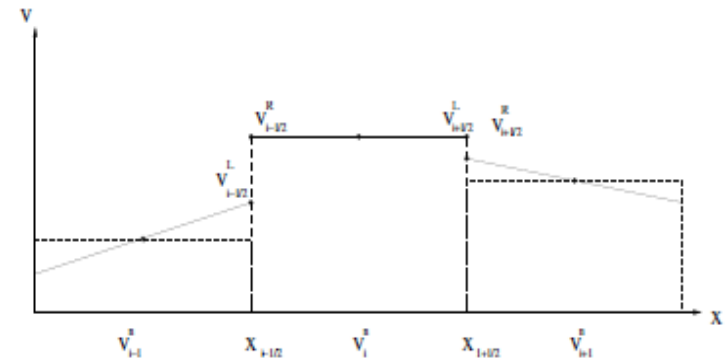
MUSCL-Hancock reconstruction: The MUSCL(Monotone Upstream Centered Schemes for Conservation Laws) introduced by Van leer with PLM reconstruction.

Replace the constant piecewise value of the cell variable by a linear slope to define a newer left and right states for each cell in order to compute the inter cell fluxes with more accuracy.

$$U_i^{t,L,R} = U_i^t \pm \frac{\Delta x}{2} \zeta_i$$

$$\zeta_i = \min \text{mod}(U_{i-1} - U_i, U_{i+1} - U_i)$$

$$U_i^{t+1/2,L,R} = U_i^{s,L,R} - \frac{\Delta t}{2\Delta x} (F(U_i^{t,R}) - F(U_i^{t,L}))$$



$$U_i^t = U_i^t - \frac{\Delta t}{\Delta x} (F(U_i^{t+1/2,R}, U_{i+1}^{t+1/2,L}) - F(U_{i-1}^{t+1/2,R}, U_i^{t+1/2,L}))$$

To preserve the monotonic solution and avoid oscillations We use the Total Variation Diminishing (TVD) with minmod slope limiter. This algorithm is the MUSCL Hancock method which achieve a second order extension.

Modeling and Simulations

TVD MUSCL-Hancock reconstruction: development for 2D example

MUSCL 2D Variable reconstruction

$$\begin{aligned}
 U_{i,j}^{-x} &= U_{i,j}^n - \frac{1}{2} \eta \Delta_i & U_{i,j}^{+x} &= U_{i,j}^n + \frac{1}{2} \eta \Delta_i \\
 U_{i,j}^{-y} &= U_{i,j}^n - \frac{1}{2} \eta \Delta_j & U_{i,j}^{+y} &= U_{i,j}^n + \frac{1}{2} \eta \Delta_j
 \end{aligned}$$

Evolution

$$\begin{aligned}
 \zeta_{i,j}^{-X} &= U_{i,j}^{-X} + \frac{1}{2} \frac{\Delta t}{\Delta x} \left(Fx(U_{i,j}^{-X}) - Fx(U_{i,j}^{+X}) \right) + \frac{1}{2} \frac{\Delta t}{\Delta y} \left(Fy(U_{i,j}^{-Y}) - Fy(U_{i,j}^{+Y}) \right) \\
 \zeta_{i,j}^{+X} &= U_{i,j}^{+X} + \frac{1}{2} \frac{\Delta t}{\Delta x} \left(Fx(U_{i,j}^{-X}) - Fx(U_{i,j}^{+X}) \right) + \frac{1}{2} \frac{\Delta t}{\Delta y} \left(Fy(U_{i,j}^{-Y}) - Fy(U_{i,j}^{+Y}) \right) \\
 \zeta_{i,j}^{-Y} &= U_{i,j}^{-Y} + \frac{1}{2} \frac{\Delta t}{\Delta x} \left(Fx(U_{i,j}^{-X}) - Fx(U_{i,j}^{+X}) \right) + \frac{1}{2} \frac{\Delta t}{\Delta y} \left(Fy(U_{i,j}^{-Y}) - Fy(U_{i,j}^{+Y}) \right) \\
 \zeta_{i,j}^{+Y} &= U_{i,j}^{+Y} + \frac{1}{2} \frac{\Delta t}{\Delta x} \left(Fx(U_{i,j}^{-X}) - Fx(U_{i,j}^{+X}) \right) + \frac{1}{2} \frac{\Delta t}{\Delta y} \left(Fy(U_{i,j}^{-Y}) - Fy(U_{i,j}^{+Y}) \right)
 \end{aligned}$$

Modeling and Simulations

TVD MUSCL-Hancock reconstruction: development for 2D example

Intercell limiters

$$U_i^L = U_i^n - \frac{1}{2}\eta\Delta_i \quad U_i^R = U_i^n + \frac{1}{2}\eta\Delta_i$$

$$\eta\Delta_i = \begin{cases} \max \left[0, \min \left(\beta\Delta_{i-\frac{1}{2}}, \Delta_{i+\frac{1}{2}} \right), \min \left(\Delta_{i-\frac{1}{2}}, \beta\Delta_{i+\frac{1}{2}} \right) \right], & \Delta_{i-\frac{1}{2}} \succ 0 \\ \min \left[0, \max \left(\beta\Delta_{i-\frac{1}{2}}, \Delta_{i+\frac{1}{2}} \right), \max \left(\Delta_{i-\frac{1}{2}}, \beta\Delta_{i+\frac{1}{2}} \right) \right], & \Delta_{i-\frac{1}{2}} \prec 0 \end{cases}$$

$$\Delta_{i-\frac{1}{2}} = U_i^n - U_{i-1}^n \quad \Delta_{i+\frac{1}{2}} = U_{i+1}^n - U_i^n$$

$$\beta = 1(\text{superbee}) \quad \beta = 2(\text{Min mod})$$

Modeling and Simulations

TVD MUSCL-Hancock reconstruction: development for 2D example

Riemann Problem solver

$$U_i^L \equiv \zeta_{i,j}^{+X} \quad U_i^R \equiv \zeta_{i+1,j}^{-X}$$

$$U_j^L \equiv \zeta_{i,j}^{+Y} \quad U_j^R \equiv \zeta_{i,j+1}^{-Y}$$

$$RP(U_i^L, U_i^R) \longrightarrow F_{i+\frac{1}{2},j} = F\left(U_{i+\frac{1}{2},j}(0)\right)$$

$$RP(U_j^L, U_j^R) \longrightarrow G_{i,j+\frac{1}{2}} = G\left(U_{i,j+\frac{1}{2}}(0)\right)$$

Update solution

$$U_{i,j,k}^{n+1} = U_{i,j,k}^n + \frac{\Delta t}{\Delta x} \left[F_{i+1/2,j,k}^{n+1/2} - F_{i-1/2,j,k}^{n+1/2} \right] + \frac{\Delta t}{\Delta y} \left[G_{i,j+1/2,k}^{n+1/2} - G_{i,j-1/2,k}^{n+1/2} \right]$$

Modeling and Simulations

Viscous terms : Finite central difference scheme is used to discretize viscous terms to compute the viscous fluxes at cell interface.

$$\frac{\partial \Theta}{\partial y}_{i+\frac{1}{2},j} = \frac{\Theta_{i+1,j} - \Theta_{i,j}}{\Delta x} \quad \frac{\partial \Theta}{\partial y}_{i+\frac{1}{2},j} = \frac{\Theta_{i,j+1} + \Theta_{i+1,j+1} - \Theta_{i,j-1} - \Theta_{i+1,j-1}}{4\Delta y}$$

Turbulence

LES : Large eddy simulation is used for the turbulent flow, by computing the eddy viscosity.(Smagorinsky model)

$$\sigma_{ij} = (\mu_{lam} + \mu_t) * \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right)$$

$$\mu_t = (C_s \bar{\Delta})^2 |\bar{S}| \quad S_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \quad \bar{\Delta} = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}, \quad \bar{S} = \sqrt{2S_{ij}S_{ij}}$$

Modeling and Simulations

Time step : the time step is computed by multiplying the courant number by the physical number to maintain the wave in the computation cell.

$$\Delta t = C_{cfl} * \min_{i,j,k} \left[\frac{\Delta x_{i,j,k}}{S_{i,j,k}^{n,x}}, \frac{\Delta y_{i,j,k}}{S_{i,j,k}^{n,y}}, \frac{\Delta z_{i,j,k}}{S_{i,j,k}^{n,z}} \right]$$

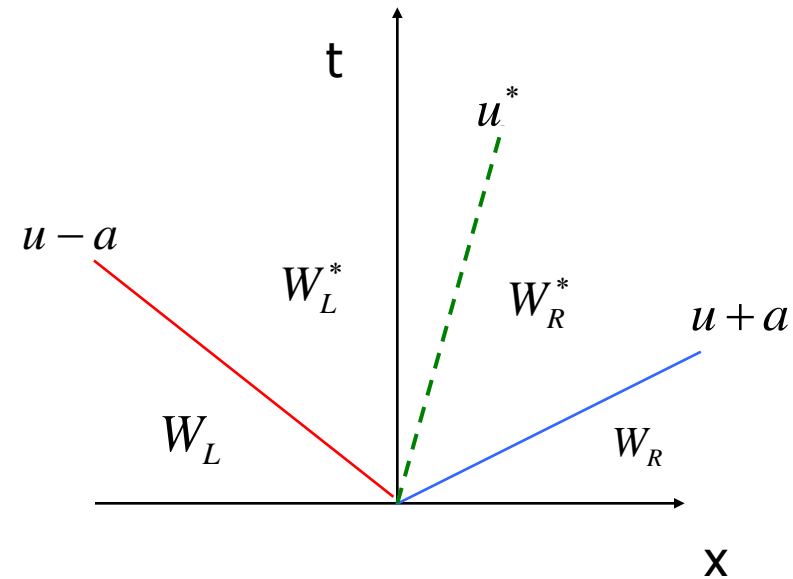
$$S_{i,j,k}^{n,x} = \sqrt{\frac{\gamma P_{i,j,k}^{n,x}}{\rho_{i,j,k}^{n,x}} + \left| (u, v, w)_{i,j,k}^{n,x} \right|}$$

Modeling and Simulations

Riemann Solver : Two fluids with different initial states Left L and Right R, interact with each other and produce two kinds of waves, shock wave and rarefaction wave in a region called star region where pressure and velocity are constant.

W is the primitive physical value

$$W = \begin{pmatrix} \rho \\ u \\ v \\ w \\ P \end{pmatrix} = \begin{pmatrix} \text{Density} \\ x\text{-velocity} \\ y\text{-velocity} \\ z\text{-velocity} \\ \text{Pressure} \end{pmatrix}$$



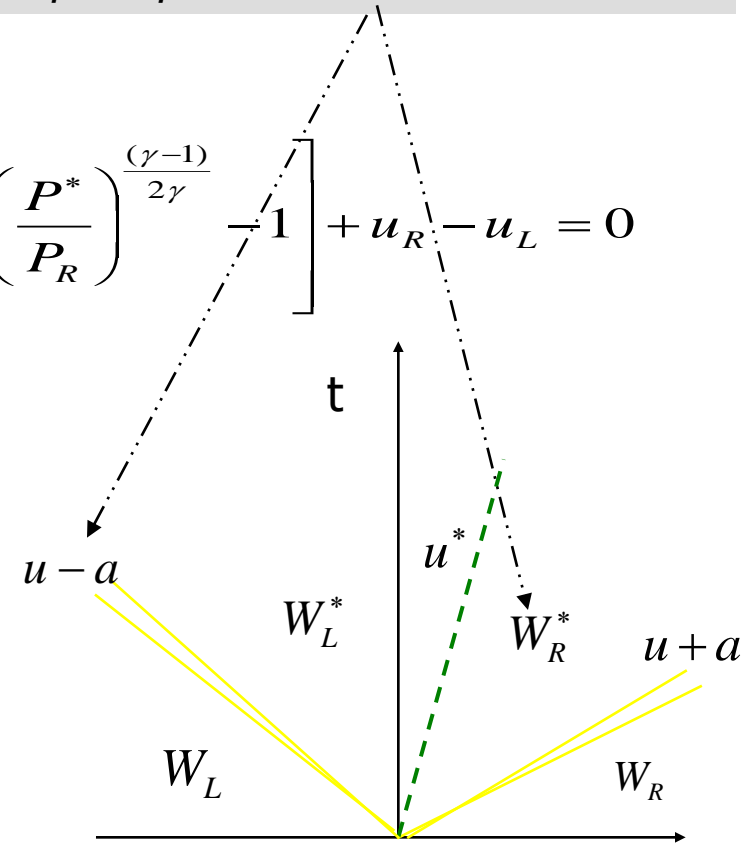
Modeling and Simulations

Two Rarefaction Riemann Solver : We assume *a priori* that both non-linear waves are *Rarefactions* which gives a more simple equation for P^* to resolve.

$$\frac{2a_L}{(\gamma-1)} \left[\left(\frac{P^*}{P_L} \right)^{\frac{(\gamma-1)}{2\gamma}} - 1 \right] + \frac{2a_R}{(\gamma-1)} \left[\left(\frac{P^*}{P_R} \right)^{\frac{(\gamma-1)}{2\gamma}} - 1 \right] + u_R - u_L = 0$$

$$P^* = \left[\frac{a_L + a_R - \frac{\gamma-1}{2}(u_R - u_L)}{\frac{a_L}{P_L^{2\gamma}} + \frac{a_R}{P_R^{2\gamma}}} \right]^{\frac{2\gamma}{\gamma-1}}$$

$$u^* = u_k - \frac{2a_k}{\gamma-1} \left[\left(\frac{P^*}{P_k} \right)^{\frac{\gamma-1}{2\gamma}} - 1 \right]$$



Modeling and Simulations

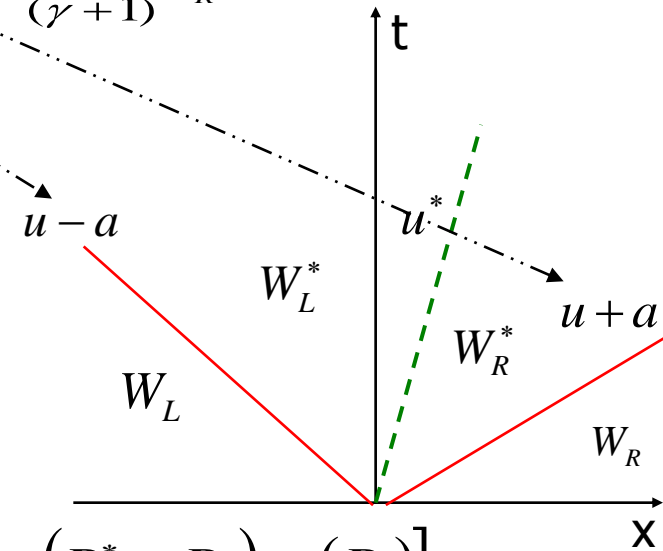
Two Shock Riemann Solver : We assume *a priori* that both non-linear waves are **Shocks** which gives a more simple equation for P^* to resolve.

$$(P^* - P_L) \sqrt{\frac{2}{\frac{(\gamma+1)\rho_L}{P^* + \frac{(\gamma-1)}{(\gamma+1)}P_L}}} + (P^* - P_R) \sqrt{\frac{2}{\frac{(\gamma+1)\rho_R}{P^* + \frac{(\gamma-1)}{(\gamma+1)}P_R}}} + u_R - u_L = 0$$

$$P^* = \frac{g_L(P_0)P_L + g_R(P_0)P_R - (u_R - u_L)}{g_L(P_0) + g_R(P_0)}$$

$$g_k(P) = \sqrt{\frac{2}{P + \frac{(\gamma-1)}{(\gamma+1)}P_k} \frac{(\gamma+1)\rho_k}{k}}$$

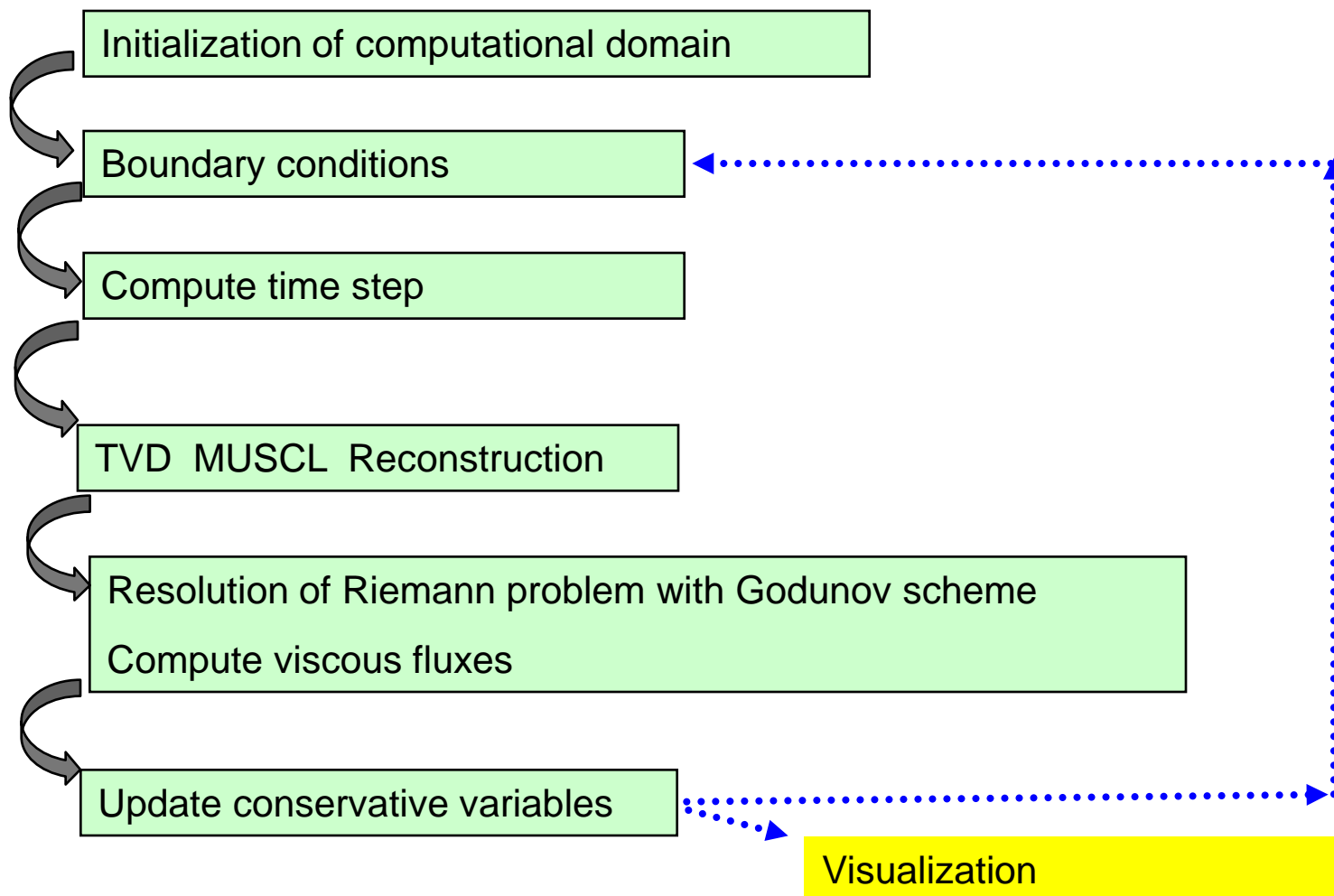
$$u^* = \frac{1}{2}(u_R + u_L) + \frac{1}{2}[(P^* - P_R)g_R(P_0) - (P^* - P_L)g_L(P_0)]$$



Modeling and Simulations

The Godunov solver : In the Godunov method, the fluxes are computed by solving Riemann problems at the interfaces between the cells. The powerful character of this approach is that the exchanges between the fluxes can be computed if the solution is initially discontinuous.

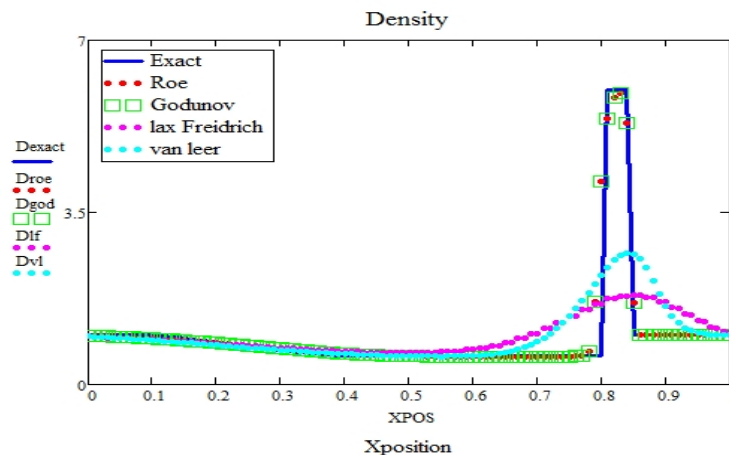
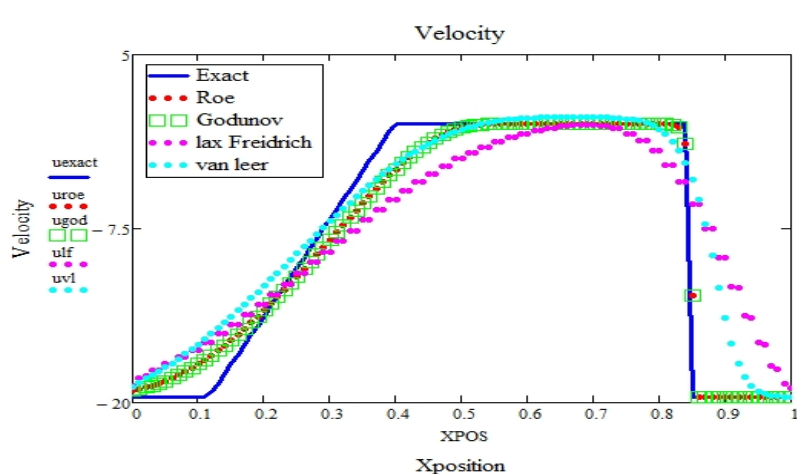
Algorithm



Results

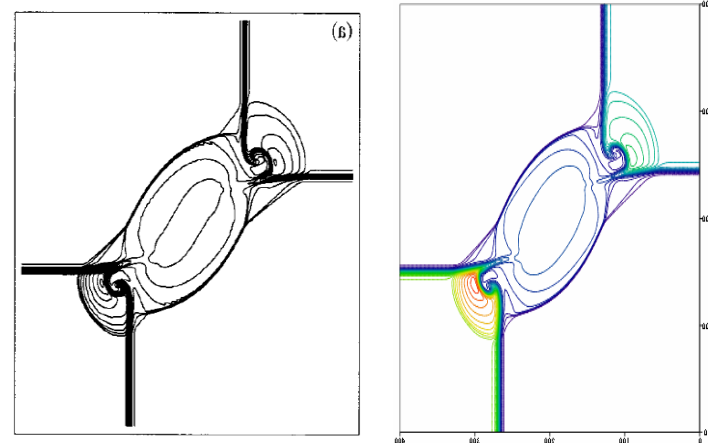
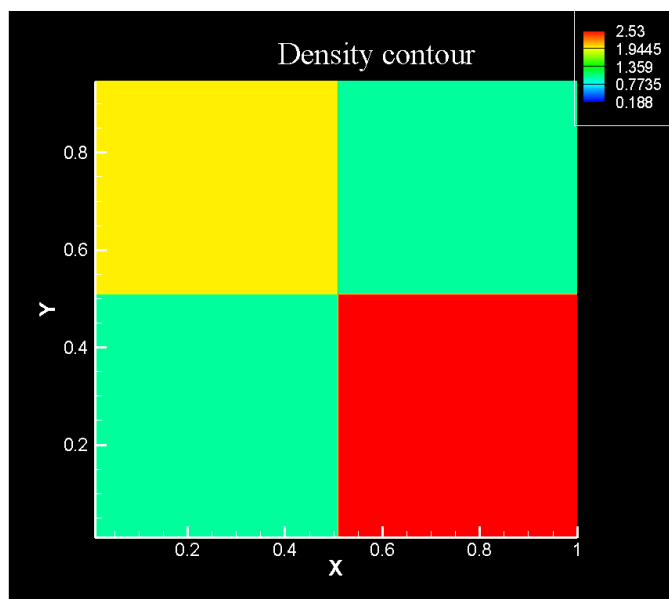
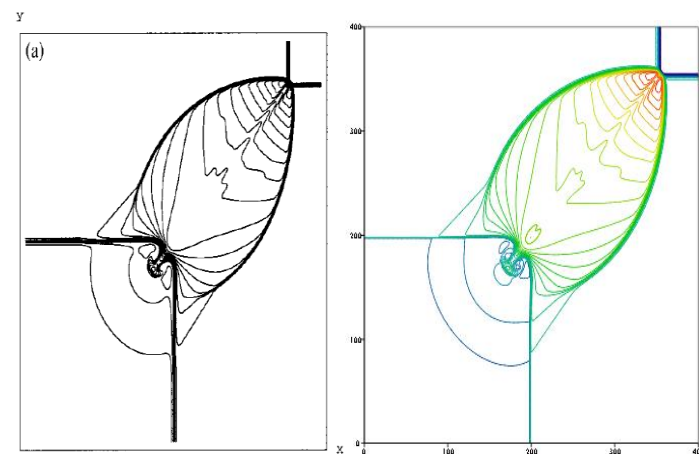
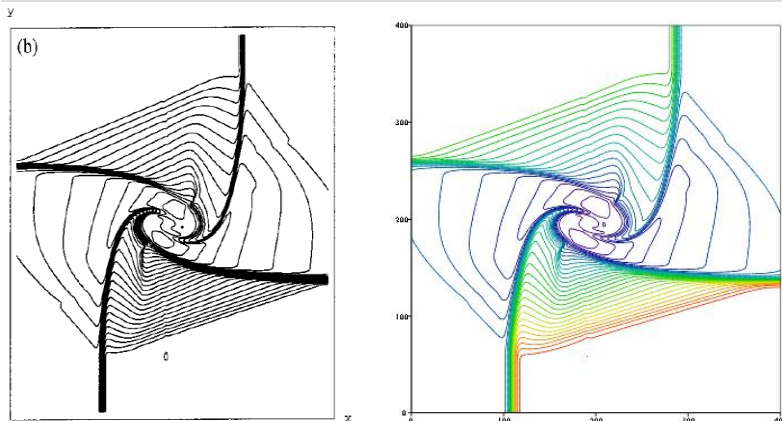
1 D Shock tube : tube separated to two compartments each one is filled with gas at different physical properties.

Test	ρ_L	u_L	P_L	ρ_R	u_R	P_R	$x0$	τ
	1.	-19.597	1000.	1.	-19.597	0.01	0.8	0.012



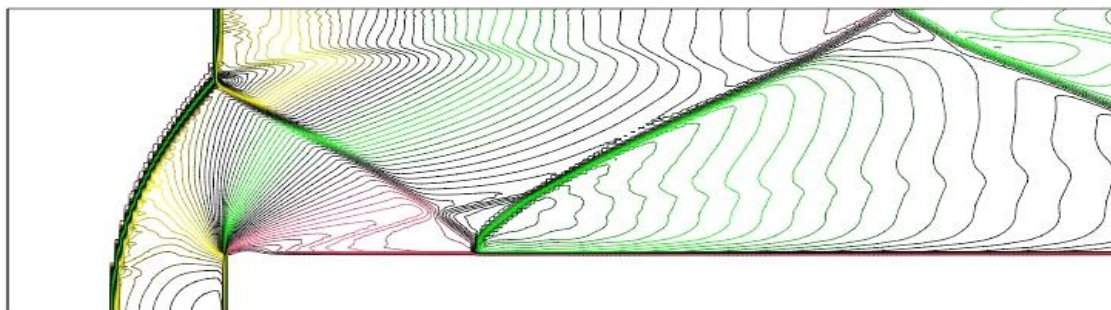
Results

2 D Shock tube : tube separated to two compartments each one is filled with gas at different physical properties.

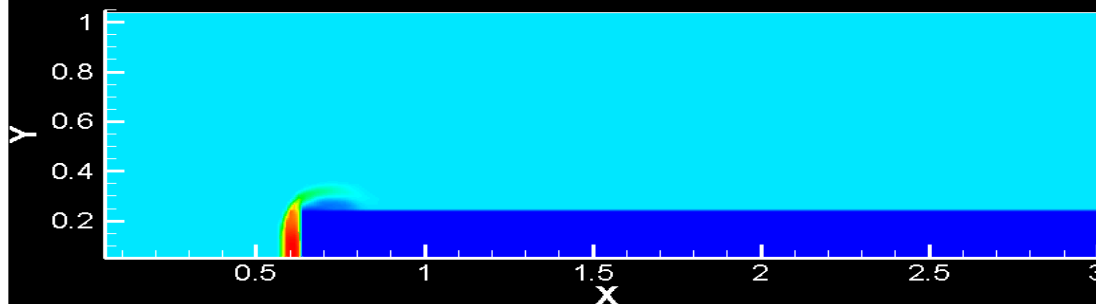
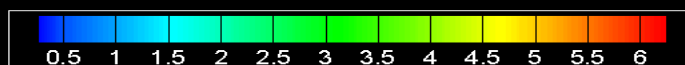


Results

Mach 3 Air tunnel : Development of density contour for a mach 3 air tunnel facing a step



Mach 3 Density contours



Results for fluid of interest : water

Does Water quasi-compressible or incompressible ?

$$\frac{\partial \rho}{\partial t} \leq 0.01 = 1\%$$

$$\frac{\partial \rho}{\partial t} = 0$$

Equation of state

Tait EOS gives a relation between pressure and density, so in the Navier Stokes equation there is no need to compute the energy equation.

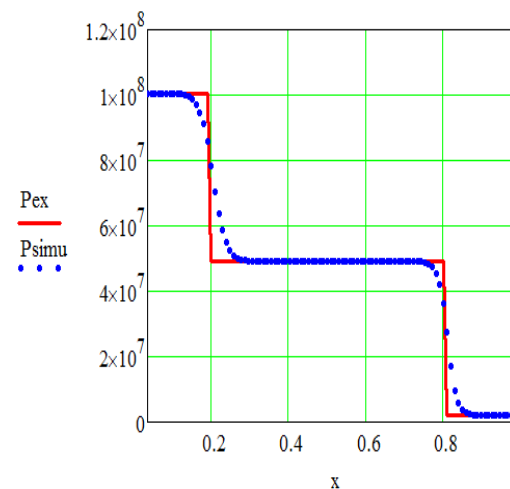
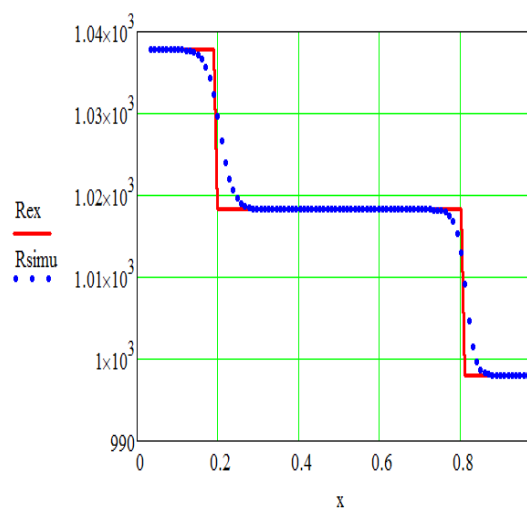
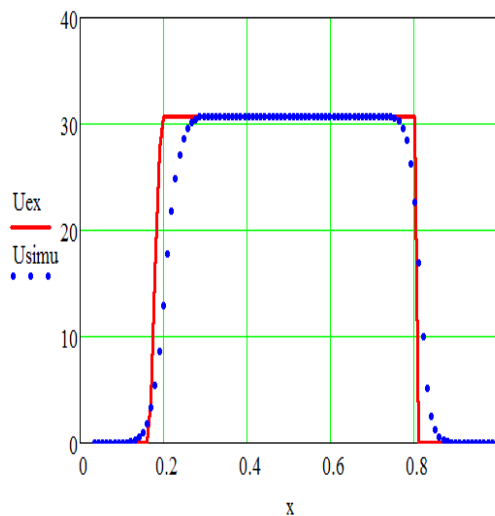
$$P = \frac{\rho c_0^2}{\gamma} \left(\left(\frac{\rho}{\rho_0} \right)^\gamma - 1 \right) + P_0$$

This pseudo compressible formulation permits an explicit treatment of time advancing method.

Results

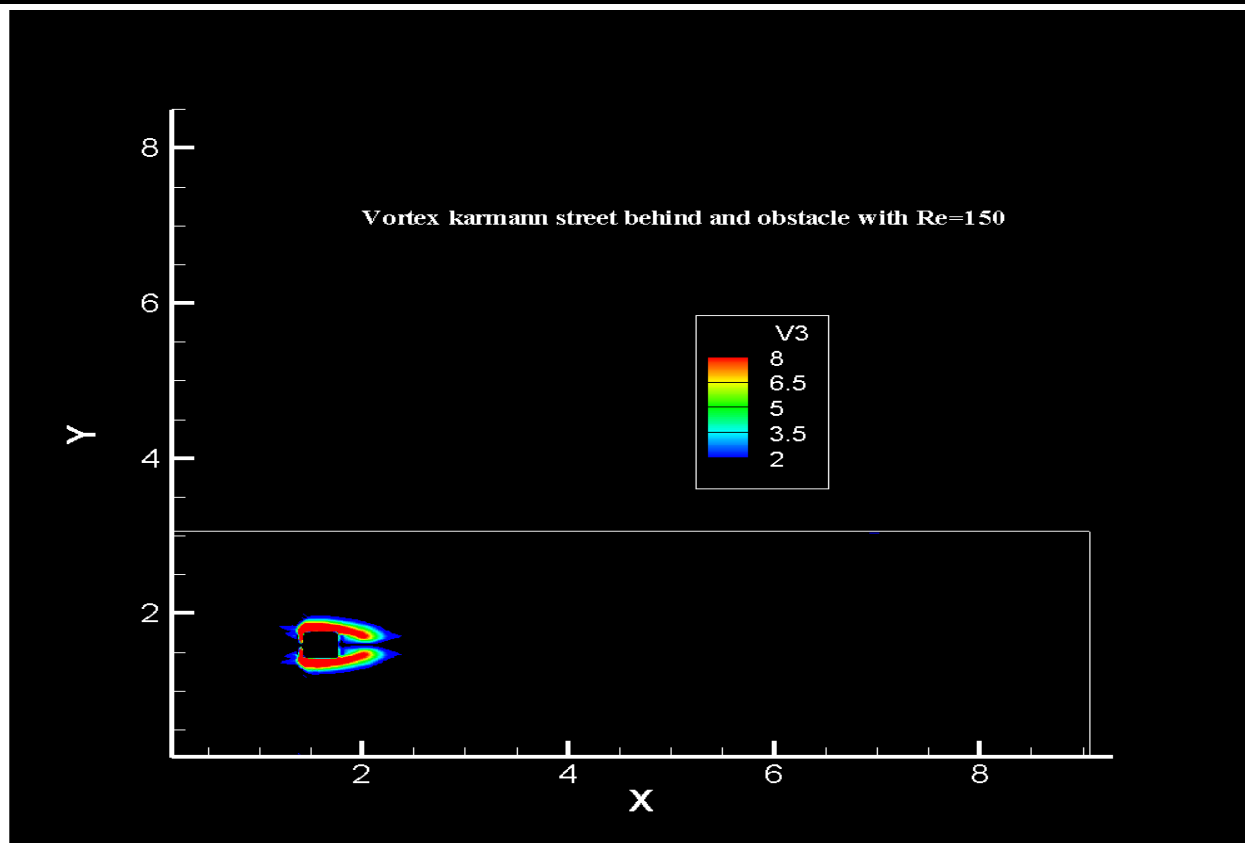
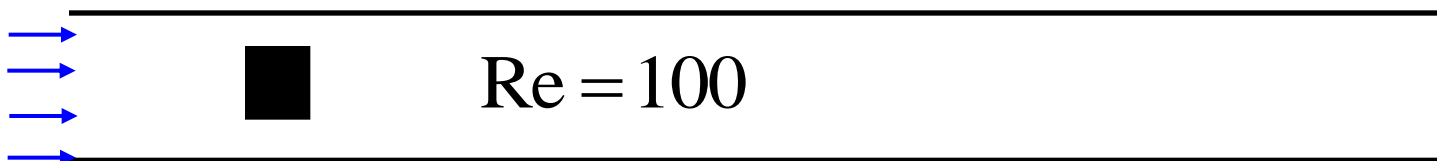
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Results for water flow

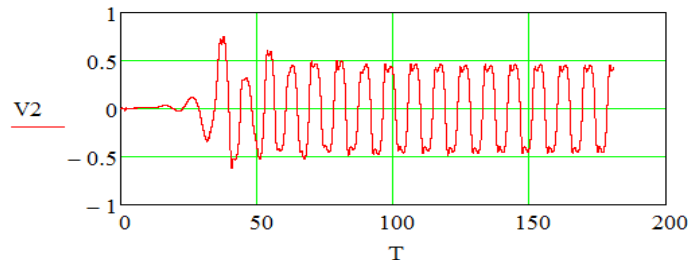
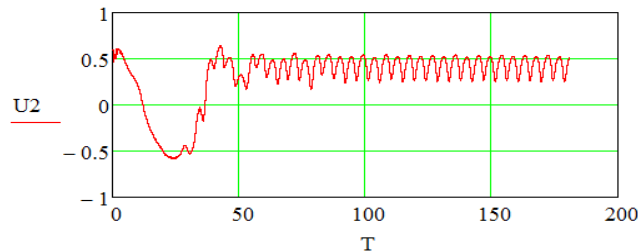
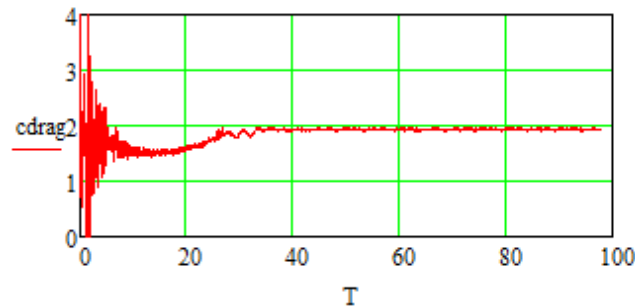
Water flow around a square cylinder.



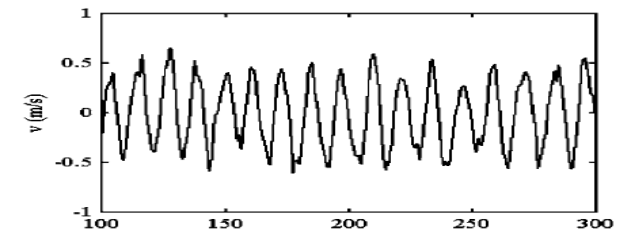
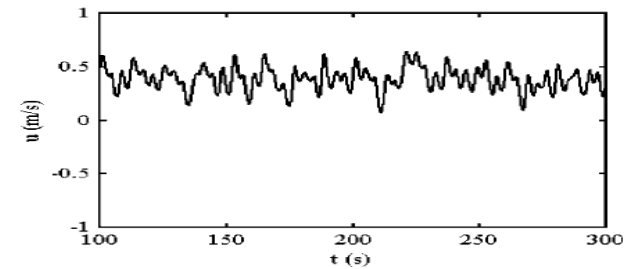
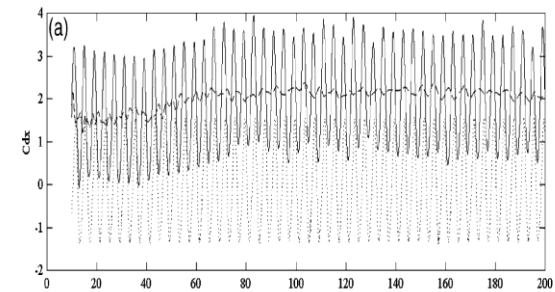
Results for water flow

Validation for 2D flow

Cd = 1.995

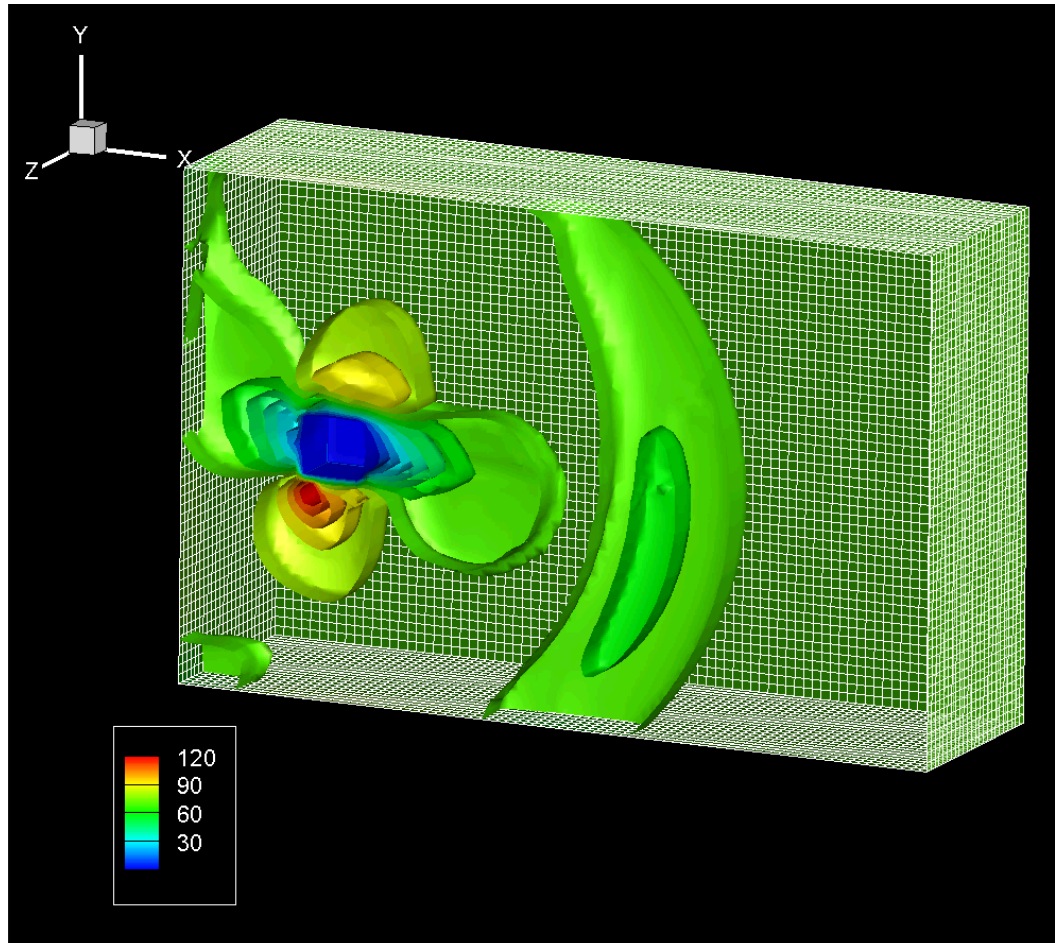


Cd = 2



Results for water flow

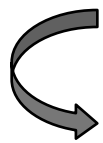
Water flow at $Re = 150$ for 3D



Results for water flow

Implantation of source terms in 2D configuration

$$U_{i,j}^{n+1} = U_{i,j}^n + \frac{\Delta t}{\Delta x} \left[F_{i+1/2,j}^i - F_{i-1/2,j}^i \right] + \frac{\Delta t}{\Delta y} \left[G_{i,j+1/2}^i - G_{i,j-1/2}^i \right]$$



$$\tilde{U}_{i+1/2,j} \quad S_{i,j} = S(U_{i,j}) \equiv \rho \vec{F}$$

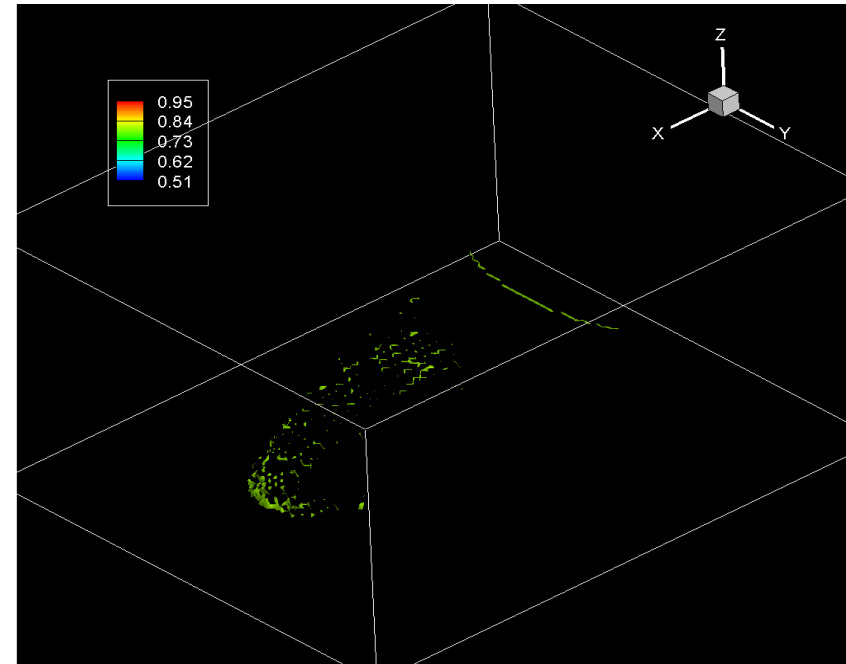
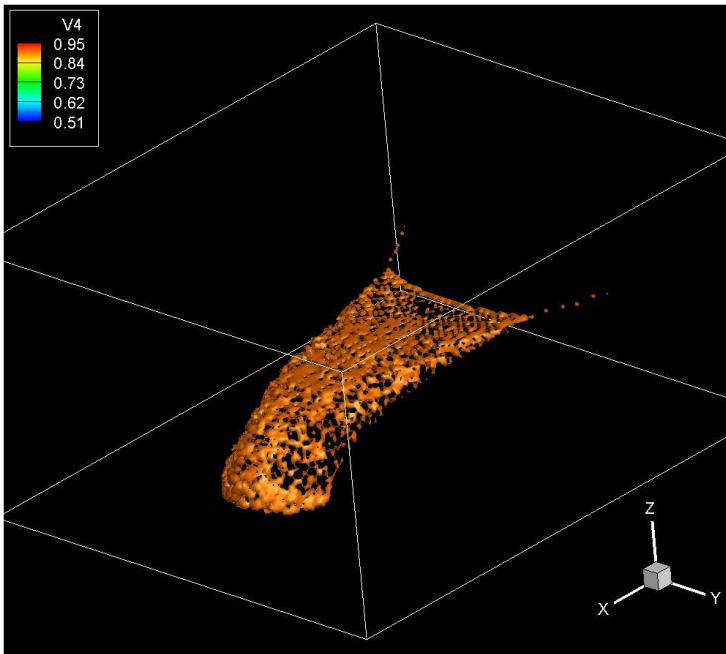


$$S_{i+1/2,j} = S(\tilde{U}_{i+1/2,j}) \equiv \rho \vec{F}(\tilde{U}_{i+1/2,j})$$

$$U_{i,j}^{n+1} = U_{i,j}^n + \frac{\Delta t}{2} \left[S_{i,j} - S_{i-1,j} \right] + \frac{\Delta t}{2} \left[S_{i,j} - S_{i,j-1} \right]$$

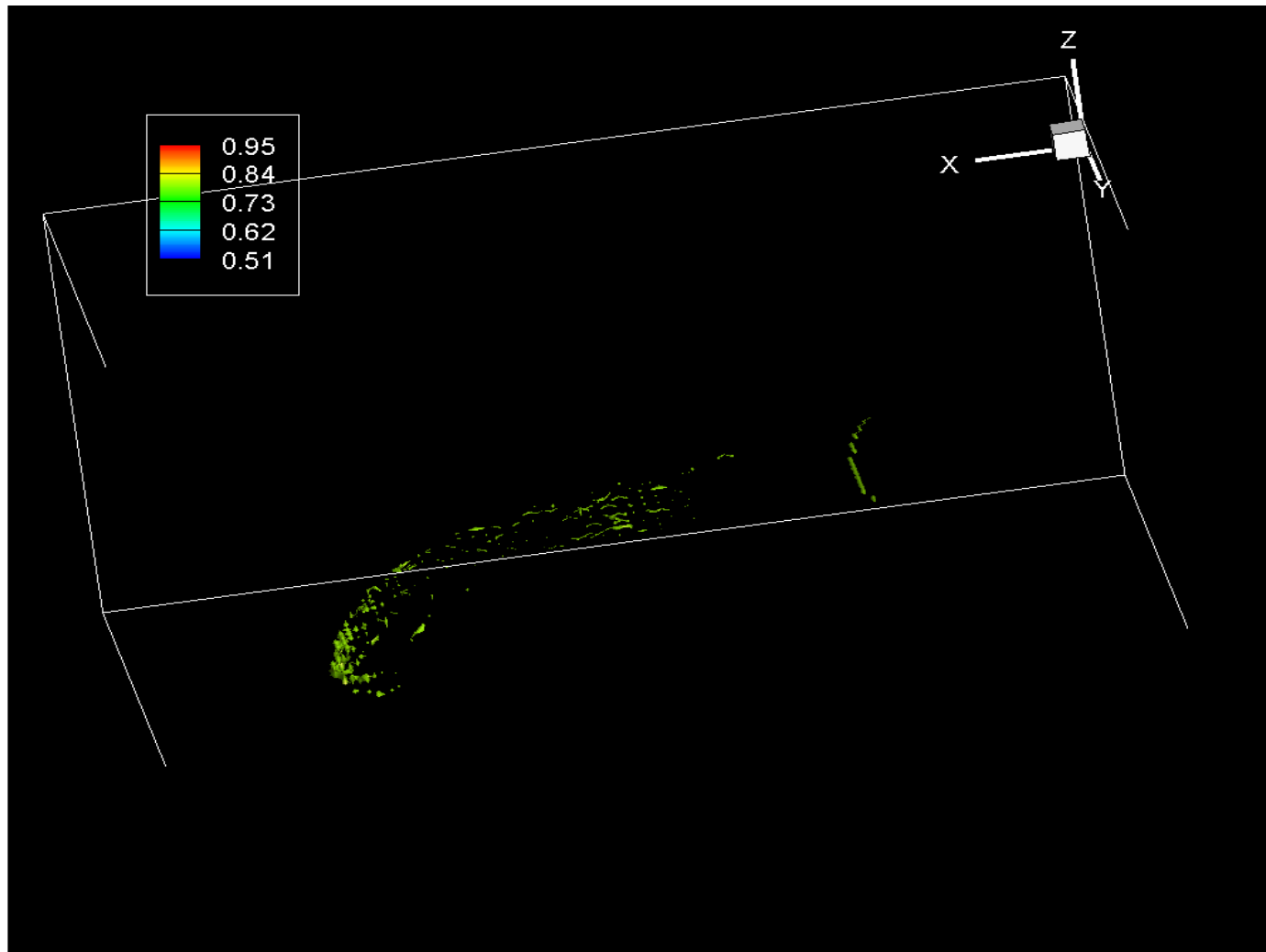
Results for water flow

Results for coupling IFREMER structure code and the fluid code



Results for water flow

Results for coupling IFREMER structure code and the fluid code



Conclusions

Main object : ameliorate the trawl net of the IFREMER net structure code by adding a fluid routine to have more accurate velocity field.

A 3D fluid dynamic code has been developed based on C++ language after being validated on one-dimensional and bi-dimensional examples.

The phase of coupling the two codes is under investigation.

Outlook

Due to the explicit method an MPI parallelization of the algorithm would be more easy and helps to reduce time computation and use more efficiently the new computer architecture

A mesh refinement near source term points would give more precision and reduce the hole time computation.